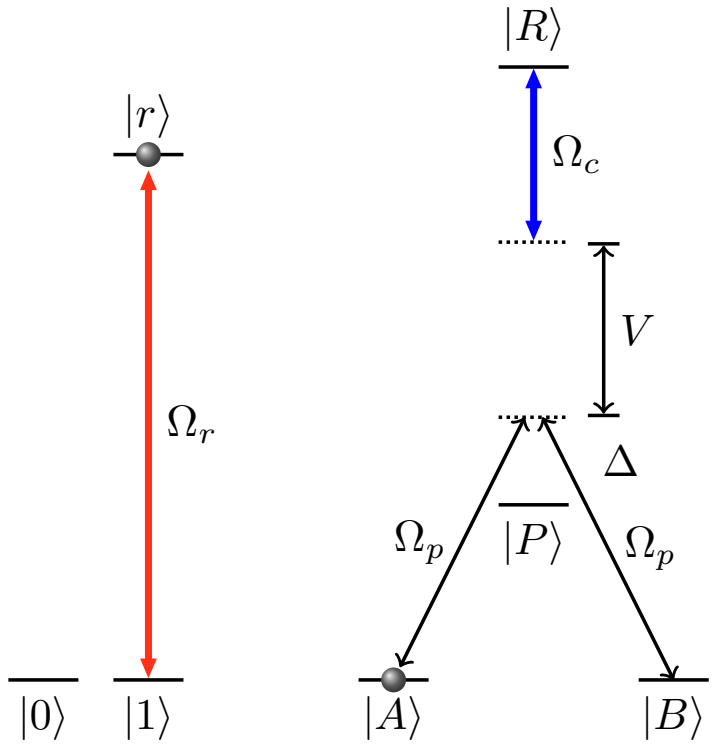
Hauptseminar: Rydberg Physics

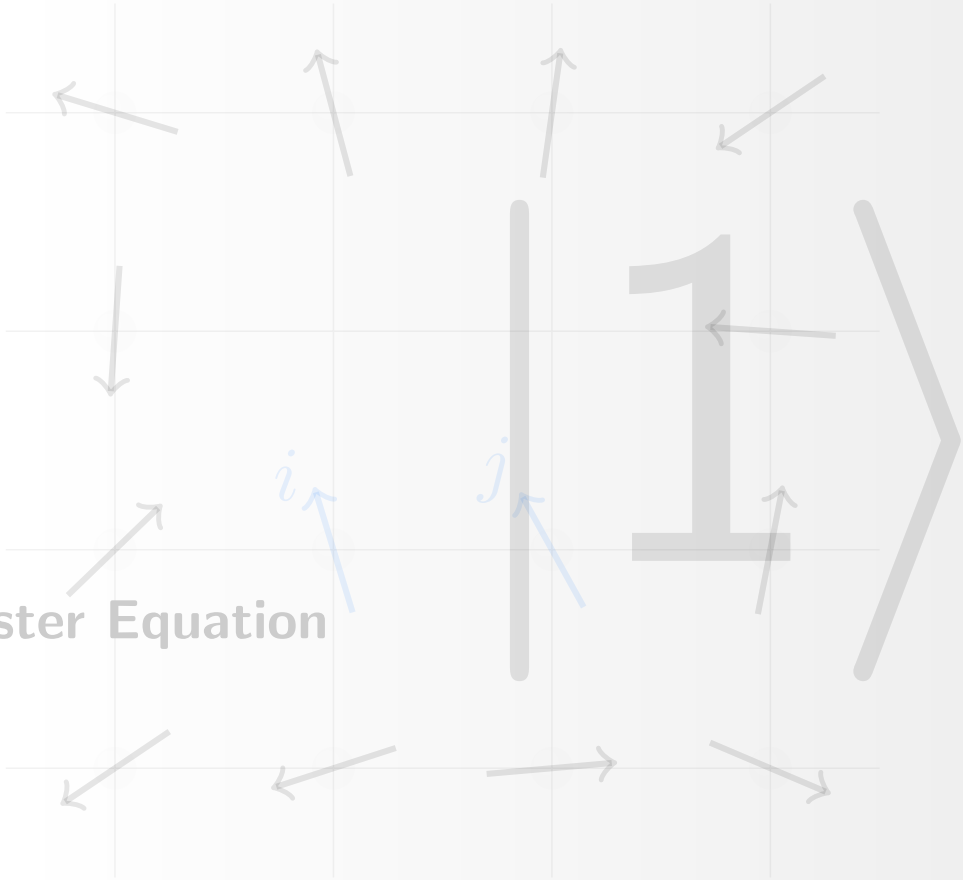


Rydberg Quantum Simulation
Ground State Preparation by Master Equation

Henri Menke — University of Stuttgart — January 29, 2015

Outline

Outline

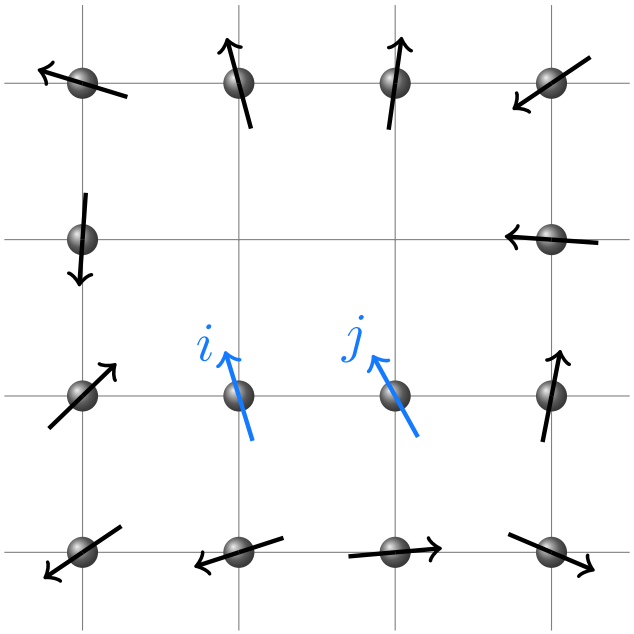


Strongly Correlated Electronic Systems

Hubbard Model

The Hamiltonian in second quantization reads

$$H = -t \sum_{\langle i,j \rangle, \sigma} c^{\dagger}_{i,\sigma} c_{j,\sigma} + U \sum_{i} n_{i,\uparrow} n_{i,\downarrow}$$



- Problems in condensed matter physics
- High-T_c superconductors, Magnets, etc.
- Not analytically solvable
- Numerically impossible for many particles

Problem

Exponential growth of the Hilbert space with the particle number.

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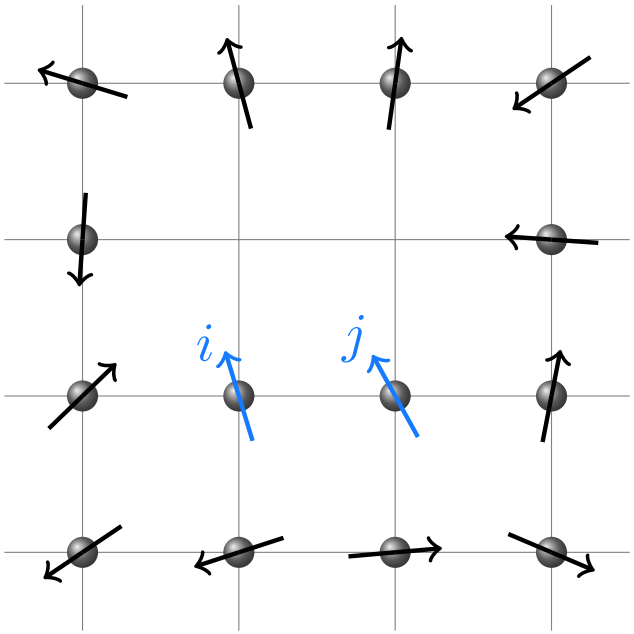

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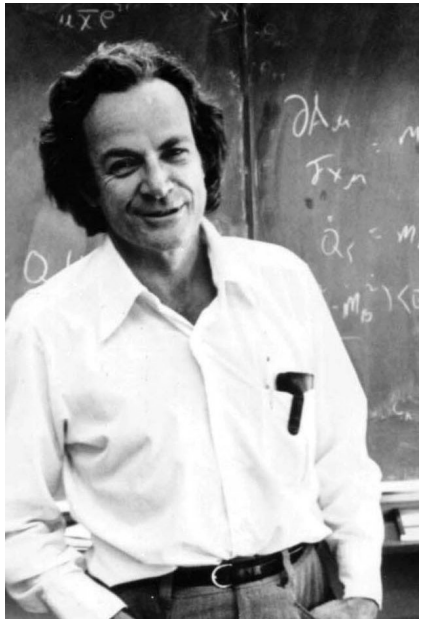
Feynman's Answer [Fey82; Llo96]

- Current state of the art: 40 particles, 2^{40} variables
- 300 particles one would require 2^{300} variables, which is the number of particles in the universe.



Simulating Physics with Computers

"Let the computer itself be built of quantum mechanical elements which obey quantum mechanical laws."

- Further elaborated by Lloyd: A Universal Quantum Simulator could simulate the dynamics of other systems with short-range interactions.

$$\exp\left(\frac{\mathrm{i}}{\hbar}\mathcal{H}t\right) \approx \exp\left(\frac{\mathrm{i}}{\hbar}\mathcal{H}_1 t\right)\exp\left(\frac{\mathrm{i}}{\hbar}\mathcal{H}_2 t\right)\cdots\exp\left(\frac{\mathrm{i}}{\hbar}\mathcal{H}_n t\right)$$

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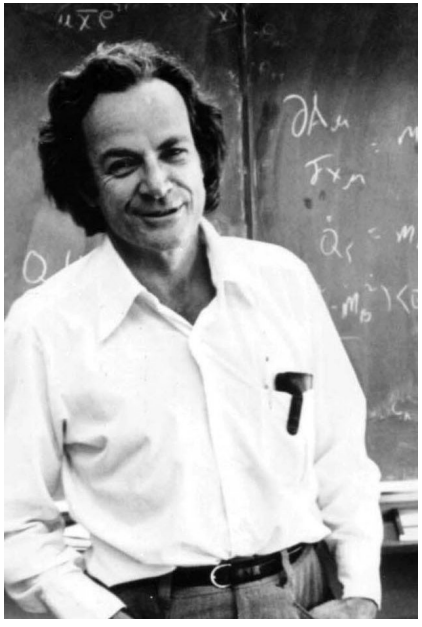

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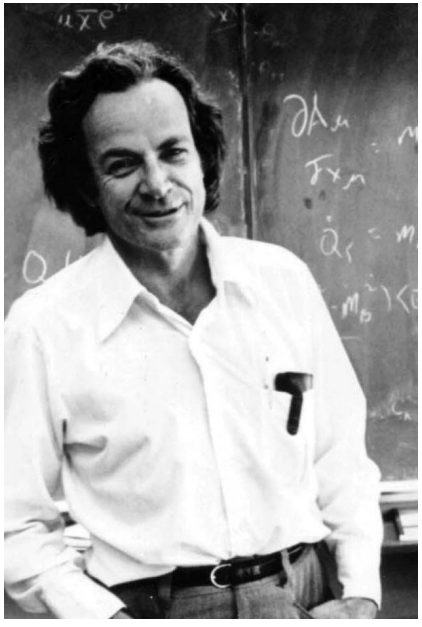

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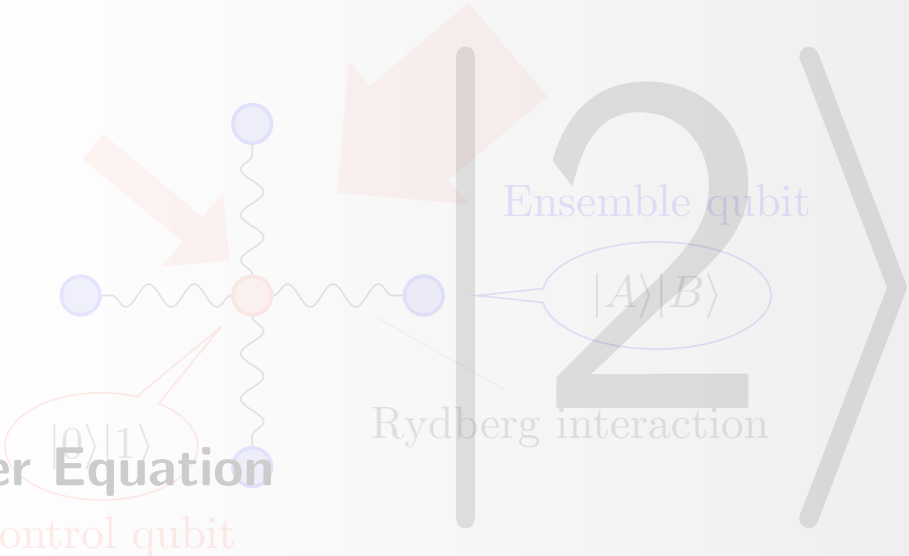
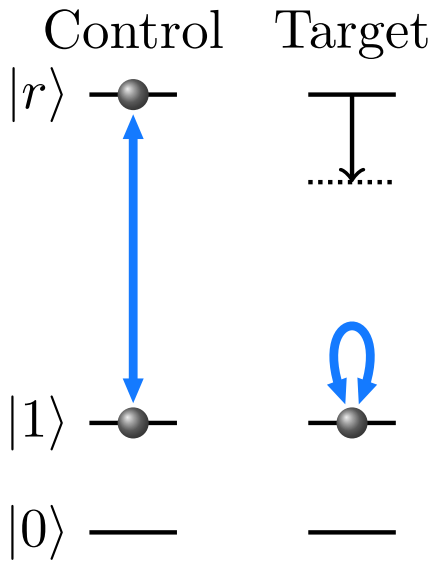


Rydberg Gates Revisited [Urb+09]

- Large dipole moment gives rise to strong Rydberg-Rydberg interaction
- The van der Waals coefficient of the repulsion scales like

$$C_6 \sim n^{11}$$



- One atom can be excited into a Rydberg state, but a second one in the vicinity cannot
- The Ryd-Ryd interaction shifts the Rydberg level of the second atom out of resonance

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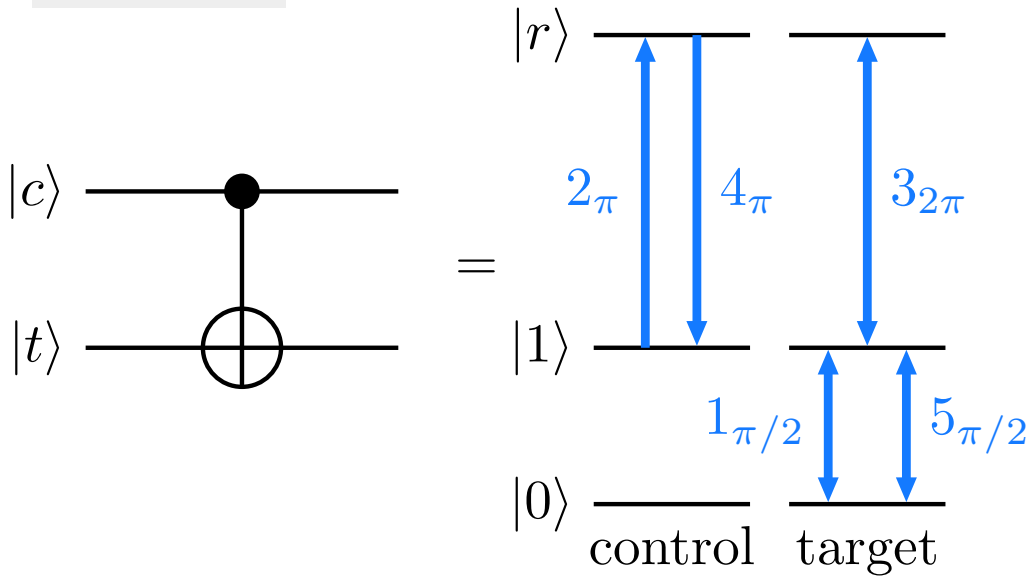

↪ talk by Niklas

Rydberg Gates [Wei10; Mül+09]

- Common setup: Atoms trapped in deep optical lattice
- Rydberg atoms possess long-range interactions
- Allows for large spacing and gives rise to better single-site addressability

Mesoscopic Gate

Coupling to many atoms in the vicinity allows to change the state of N atoms

$$\mathrm{CNOT} \to \mathrm{CNOT}^N$$

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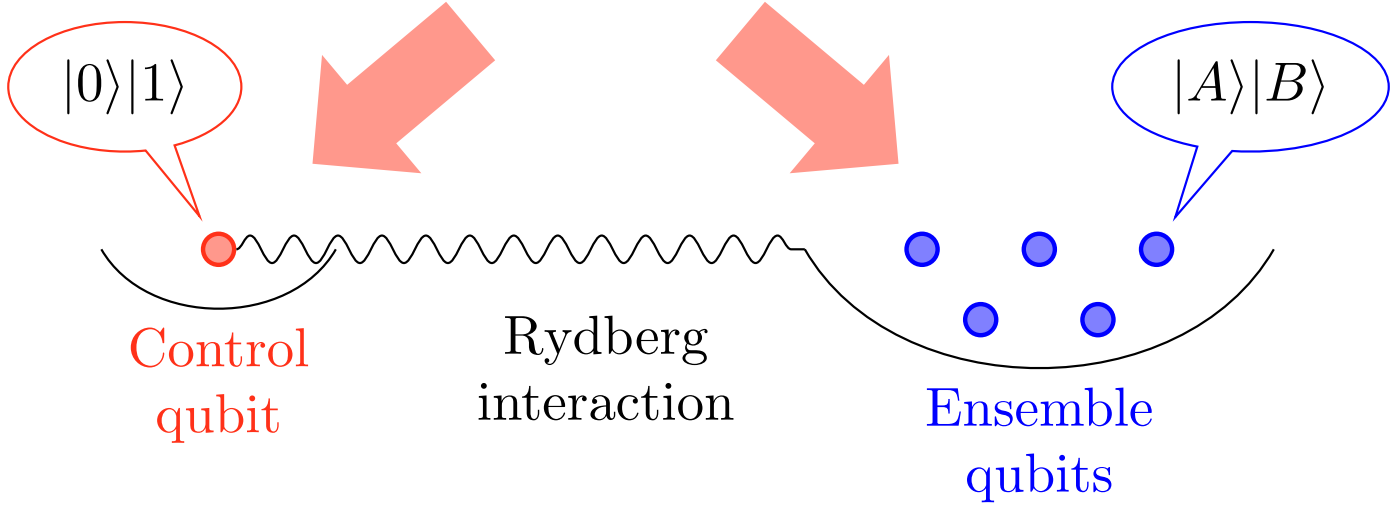

Principles of the CNOT Gate

- The CNOT gate flips the target qubit depending on the state of the control qubit

CNOT Mapping Rule

Let $|\alpha, \beta\rangle$ be a product of control and target qubit, where $\alpha \in \{0, 1\}$ denotes the control and $\beta \in \{A, B\}$ the target qubit

$$\mathrm{CNOT}\,|0, A\rangle = |0, A\rangle$$
$$\mathrm{CNOT}\,|1, A\rangle = |1, B\rangle$$

$$|0, A\rangle \to |0, A\rangle\,, \quad |1, A\rangle \to |1, B\rangle$$
$$|0, B\rangle \to |0, B\rangle\,, \quad |1, B\rangle \to |1, A\rangle$$

Mesoscopic Rydberg Gate Based on EIT [Mül+09; Wei10]

- To implement a mesoscopic CNOT gate we need to find a way to flip N qubits at once. Suppose $|A^N\rangle = \prod_i |A\rangle_i$

$$|0, A^N\rangle \to |0, A^N\rangle\,, \quad |1, A^N\rangle \to |1, B^N\rangle$$
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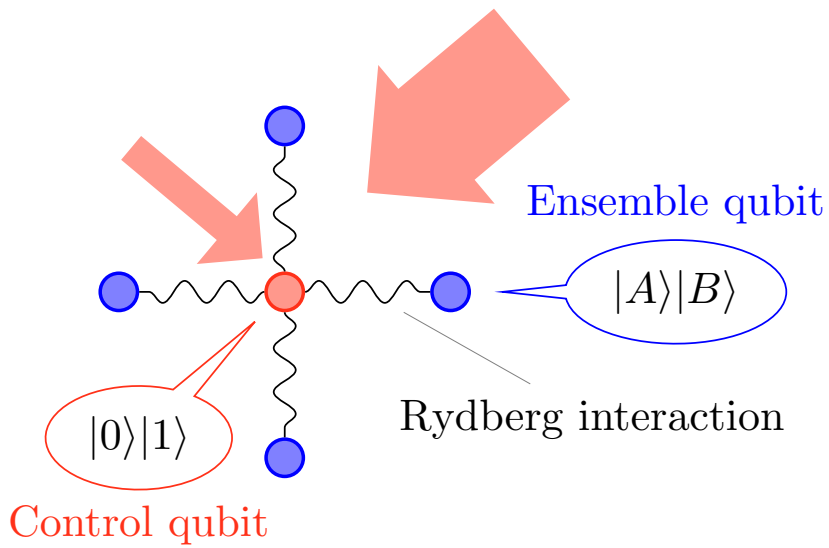
- Independent of the actual number and position of the particles
- Properly pulsed laser light drives required transitions

CNOT Gate Operator

The gate operation is unitary, so it can be easily reversed

$$U = |0\rangle\langle 0|_c \otimes \mathbb{1} + |1\rangle\langle 1|_c \otimes \prod_{i=1}^{N} \sigma_x^{(i)}$$



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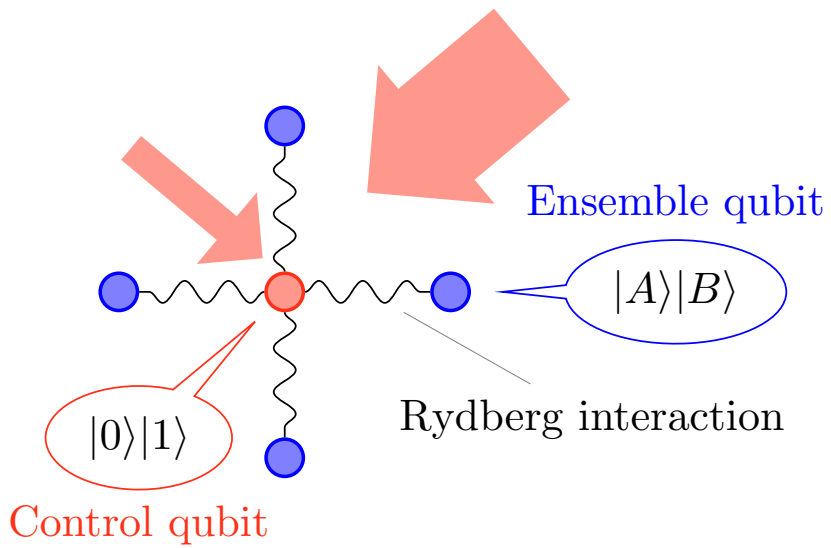

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- Control Atom in $|0\rangle$:

$$\mathrm{CNOT}\,|0, A^N\rangle = |0, A^N\rangle$$

  - EIT condition fulfilled (target is transparent for Ω_p)
  - Raman transfer is blocked

- Control Atom in $|1\rangle$:

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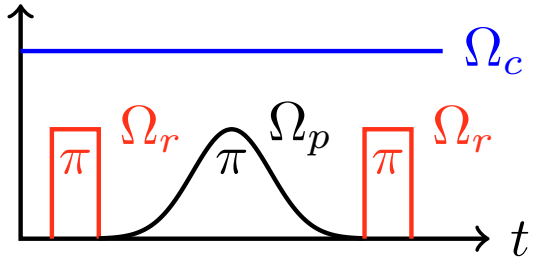

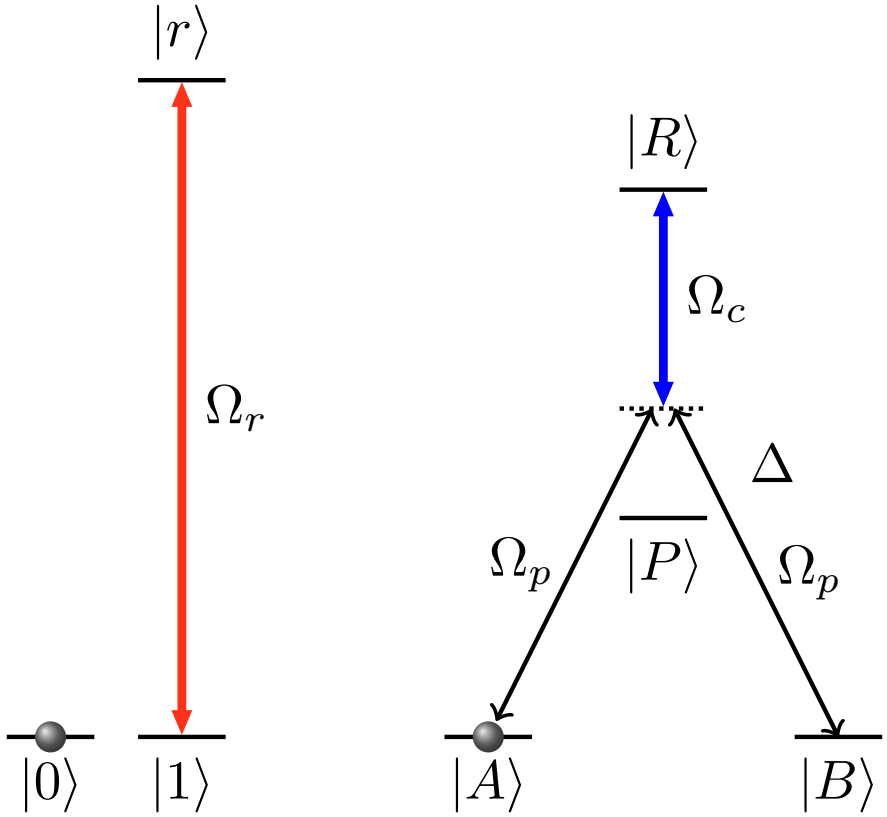

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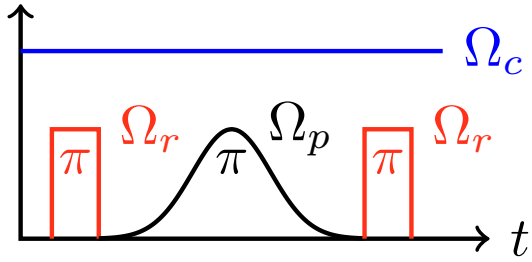
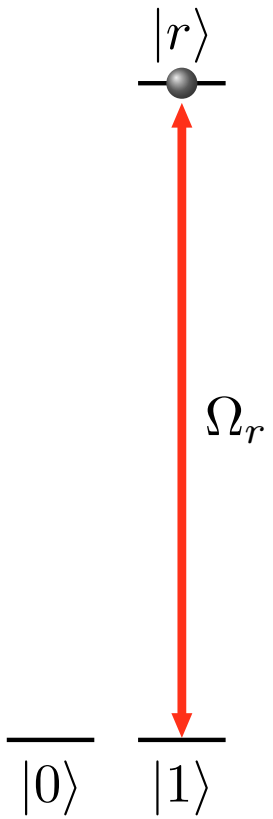
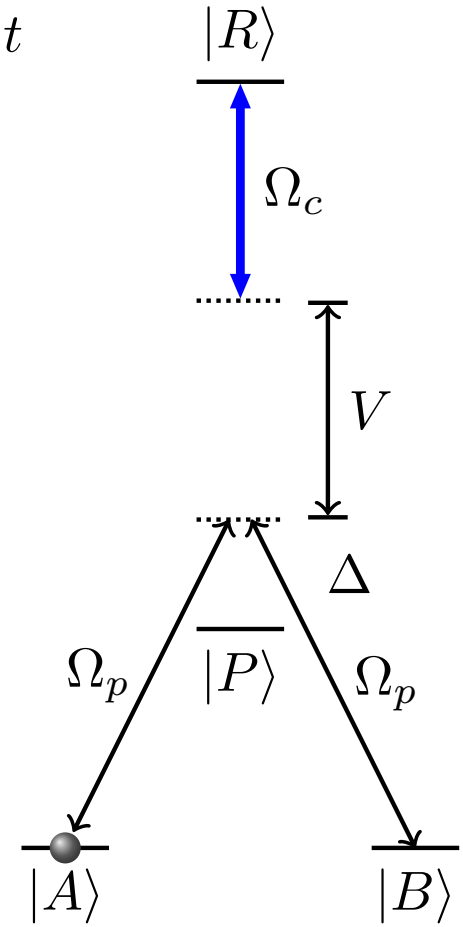






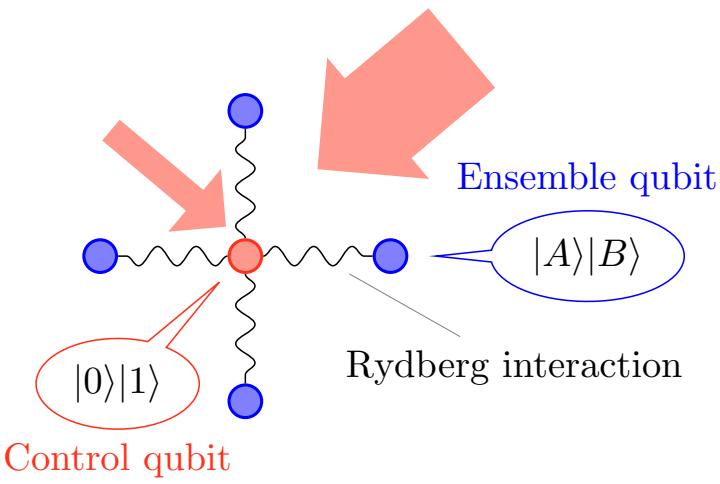
Mesoscopic Rydberg Gate Based on EIT [Mül+09; Wei10]

Many-Body Gate

We exploited long-range many-body Rydberg-Rydberg interactions to realise a many-body quantum gate

Many-Body Quantum Simulation

Can we reverse the process and simulate many-body interactions using a many-body quantum gate?



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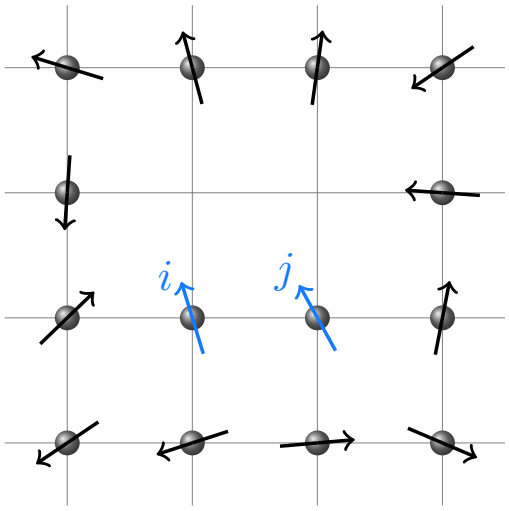

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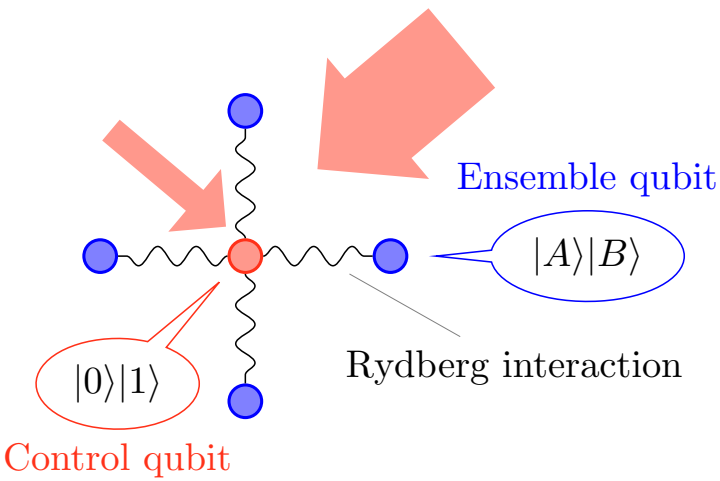

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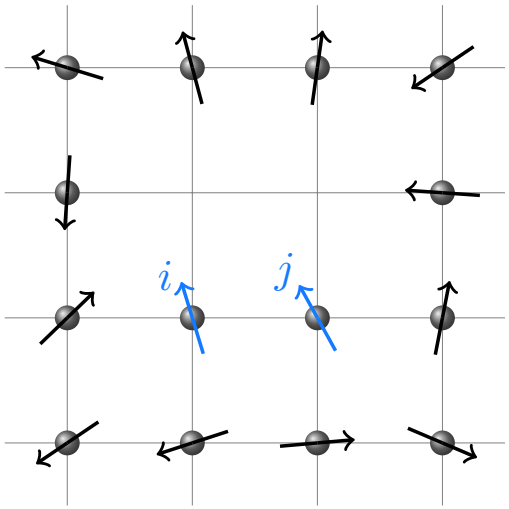

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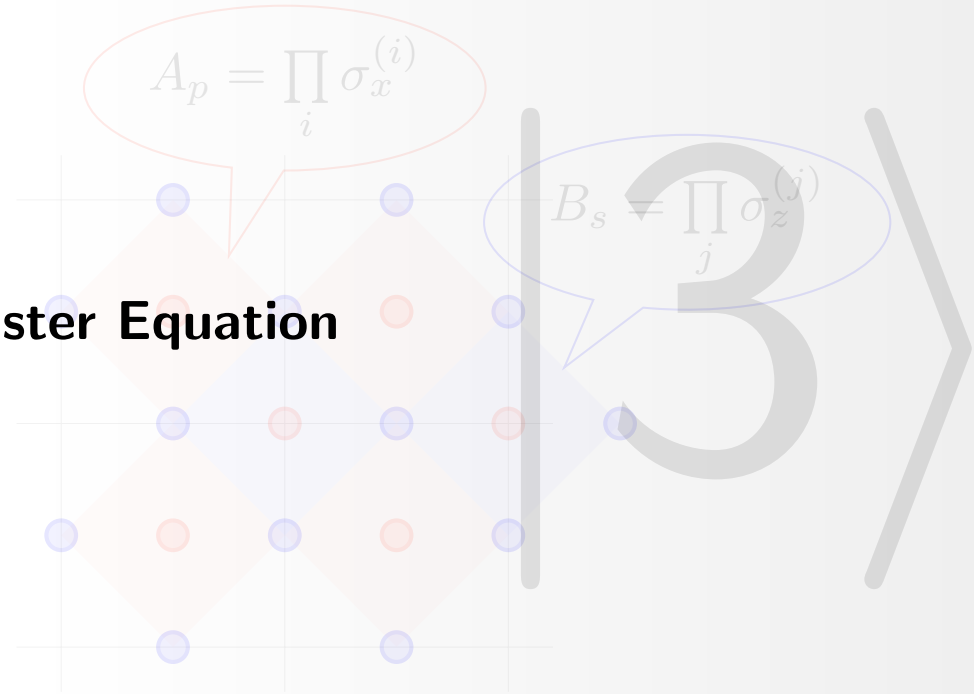
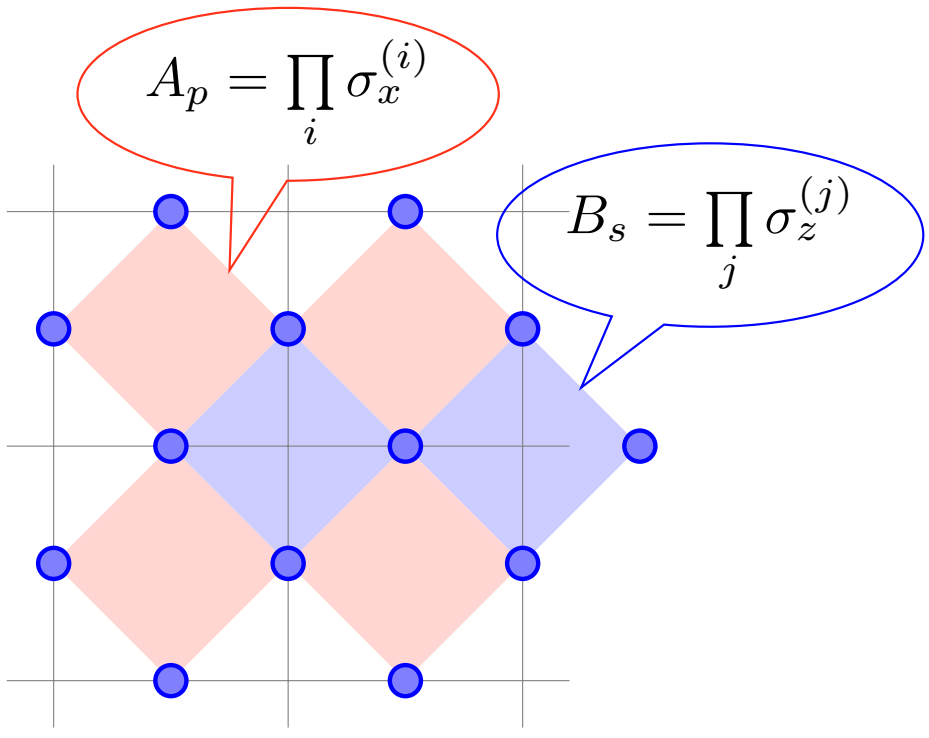


The Toric Code [Wei10; Wei+11]

- Spins are located on the edges of a two-dimensional lattice
- Two types of four-body interaction
  - Plaquette terms $A_p = \prod_i \sigma_x^{(i)}$
  - Star terms $B_s = \prod_j \sigma_z^{(j)}$



Toric Code Hamiltonian

Linear superposition of local interactions

$$\mathcal{H} = -\sum_i A_p^{(i)} - \sum_j B_s^{(j)}$$

- Global ground state $|\psi\rangle$ is eigenstate of both stabilisers

$$A_p |\psi\rangle = |\psi\rangle \ , \quad B_s |\psi\rangle = |\psi\rangle$$

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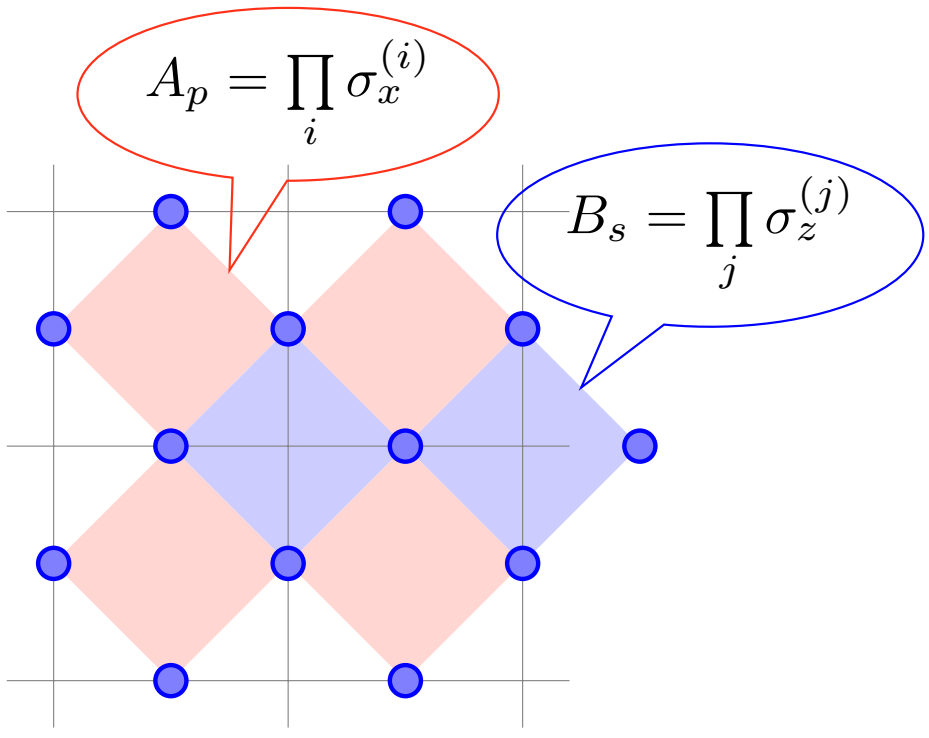

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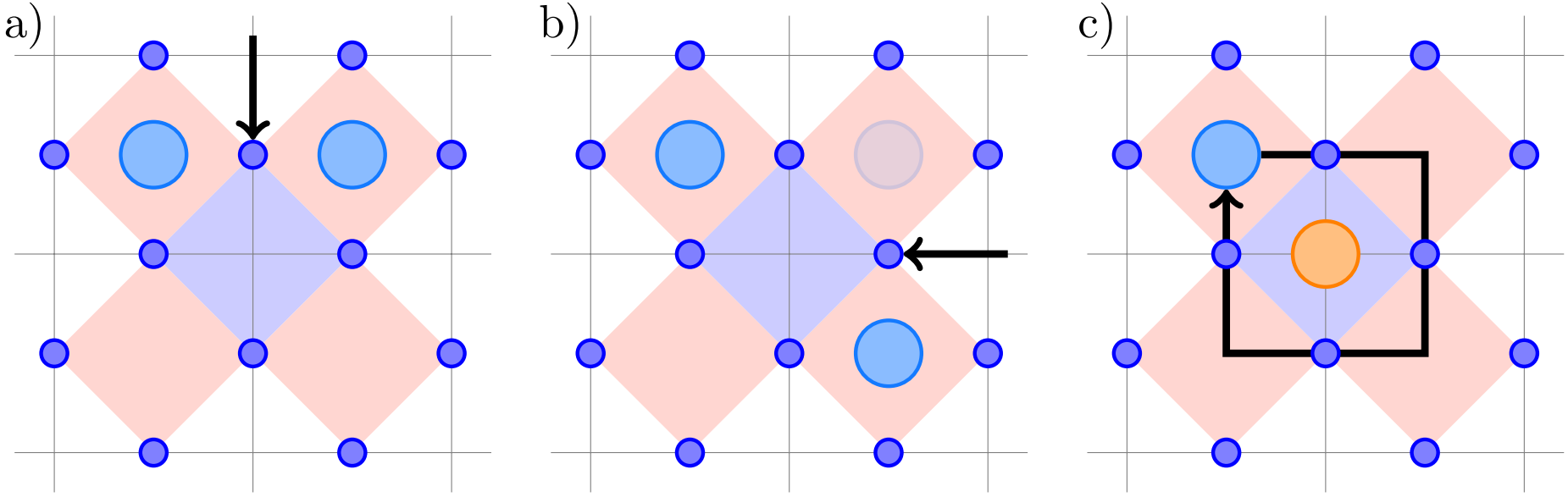
Excitations of the Toric Code [Wei10; Wei+11]

- Violations of the stabiliser constraints are called excitations
  - "Magnetic" excitation $A_p |m\rangle = -|m\rangle$
  - "Charge" excitation $B_s |e\rangle = |e\rangle$



Intermezzo: Dissipative State Preparation [BP06; Sei14]

- Dissipation is described by a coupling $V(t)$ to a heat bath
- Markovian evolution of the system

$$\varrho(t) = V(t)\varrho(0) = \mathrm{e}^{\mathcal{L}t}\varrho(0)$$

with the superoperator $\mathcal{L}$

Lindblad Master Equation

The evolution of the density matrix $\varrho(t)$ is given by a generalised Liouville-von-Neumann equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\varrho = -\frac{\mathrm{i}}{\hbar}[\mathcal{H}, \varrho] + \sum_i \gamma_i \left(c_i \varrho c_i^\dagger - \frac{1}{2}\{c_i^\dagger c_i, \varrho\} \right)$$

with jump operators c_i and decay rates γ_i.

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Dark States [BP06]

Definition: Dark State

Here we define a dark state to be a state for which all coupling to the reservoir vanishes

$$c_i \left|D\right\rangle = 0$$

- The dark state is now a stationary state of the system and a trivial solution to the master equation is

$$\varrho = \left|D\right\rangle\left\langle D\right|$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\varrho = -\frac{\mathrm{i}}{\hbar}\left(\mathcal{H}\left|D\right\rangle\left\langle D\right| - \left|D\right\rangle\left\langle D\right|\mathcal{H}\right) + \sum_i \gamma_i \left[c_i \left|D\right\rangle\left\langle D\right| c_i^\dagger - \frac{1}{2}\left(c_i^\dagger c_i \left|D\right\rangle\left\langle D\right| + \left|D\right\rangle\left\langle D\right| c_i^\dagger c_i\right)\right]$$

- Contrive a jump operator with the properties
  - The dark state is the ground state
  - The system cools itself into the ground state

State Preparation of the Toric Code [Wei10; Wei+11]

- Review: Toric Code Hamiltonian

$$\mathcal{H} = -\sum_i A_p^{(i)} - \sum_j B_s^{(j)}$$

- Jump operator for the magnetic excitations

$$c_p = \frac{1}{2}\sigma_z^{(i)}(1 - A_p)$$

- The ground state is a dark state, i.e.

$$c_p \left|\psi\right\rangle = 0$$

- The jump operator cools any density matrix into the unique ground state by
  - diffusion of excitations
  - annihilation of identical excitations

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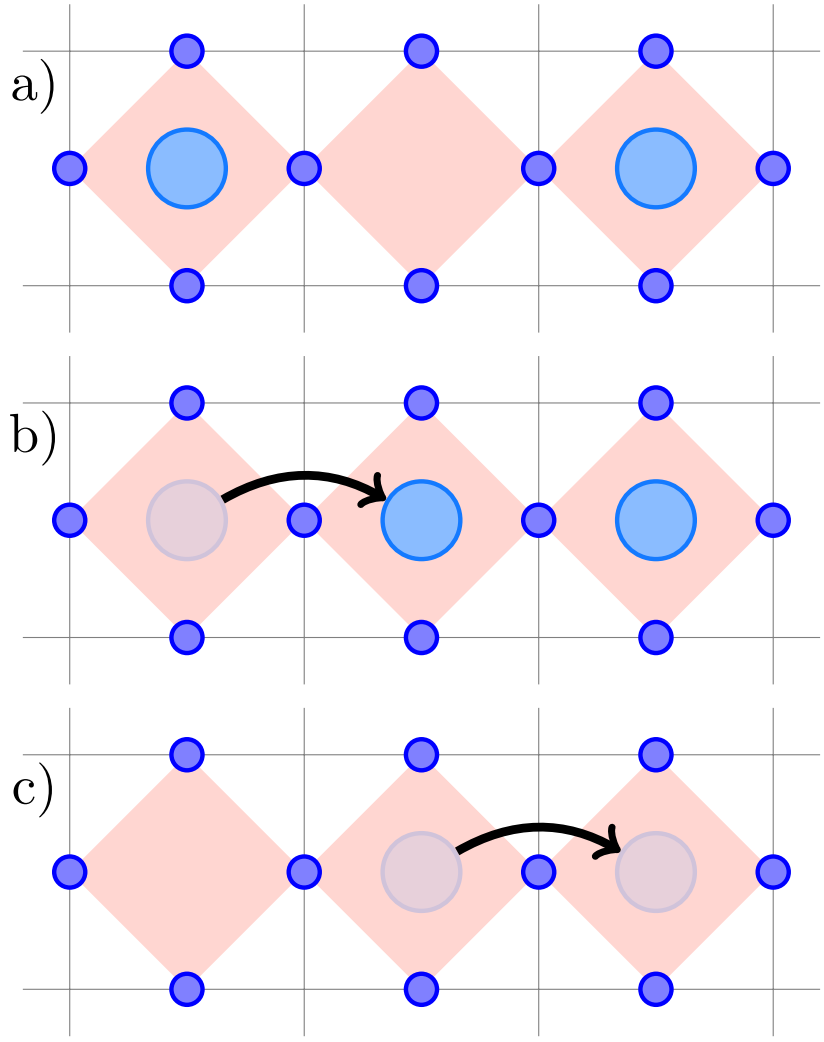

The Toric Code with Rydberg Atoms [Wei10; Wei+11]

- Rydberg atoms in a large-spacing optical lattice
- Control atoms placed in the middle of plaquettes
- Recent developments:
  - Rydberg blockade between two atoms
    Group of M. Saffman: E. Urban et al.
    Nature Physics **5**, 2 (2009), pp. 110–114
  - Selective excitation based on the Rydberg Blockade
    Group of P. Grangier: A. Gaëtan et al.
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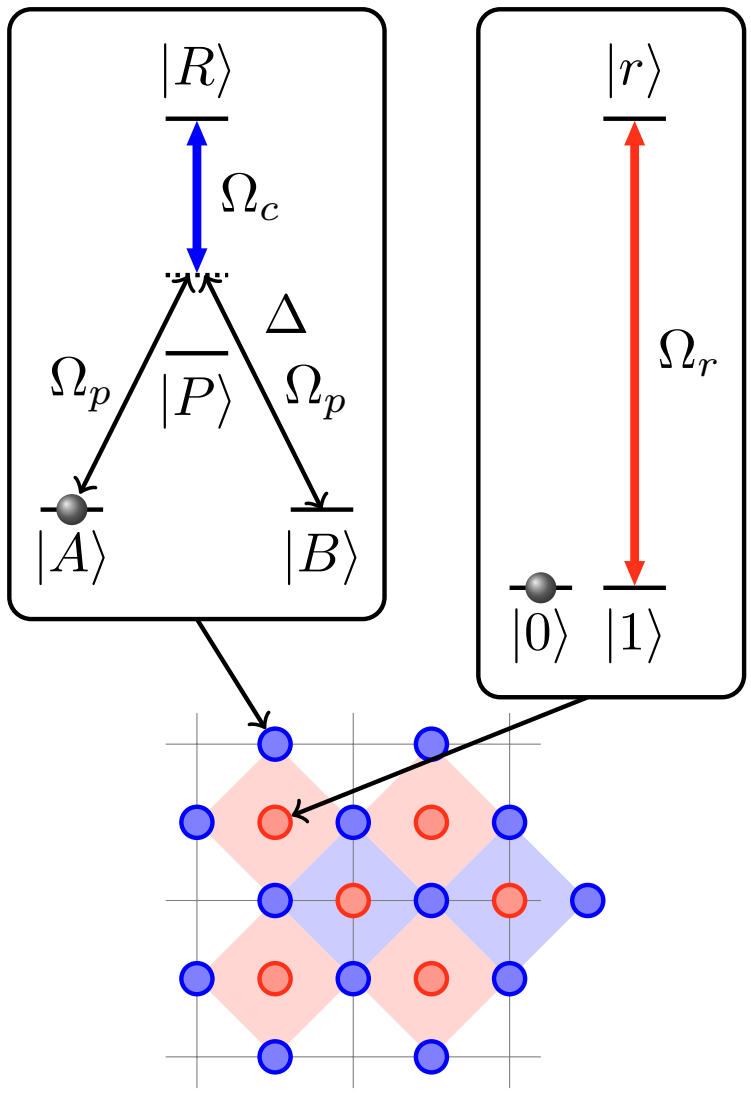
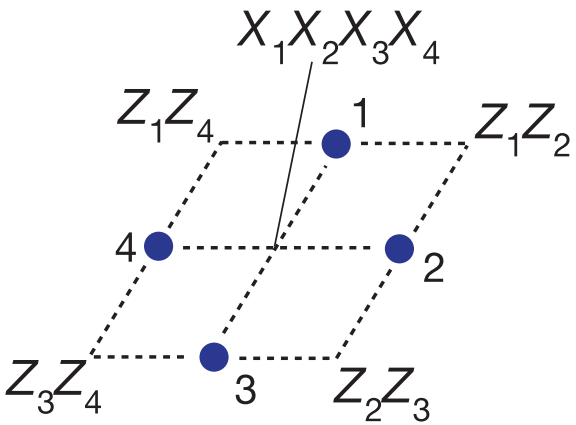
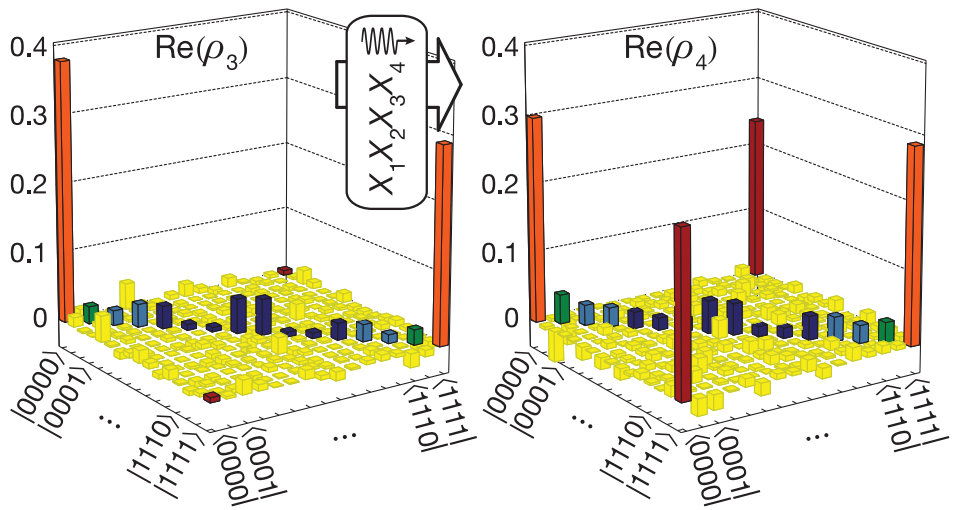


Quantum Simulation with Ultra Cold Ions [Bar+11]

- Quantum simulation with five trapped ions

  J. T. Barreiro et al. *Nature* **470**, 7335 (2011), pp. 486–491
- Minimal instance of Toric Code stabiliser
- Implements dissipative dynamics through optical pumping



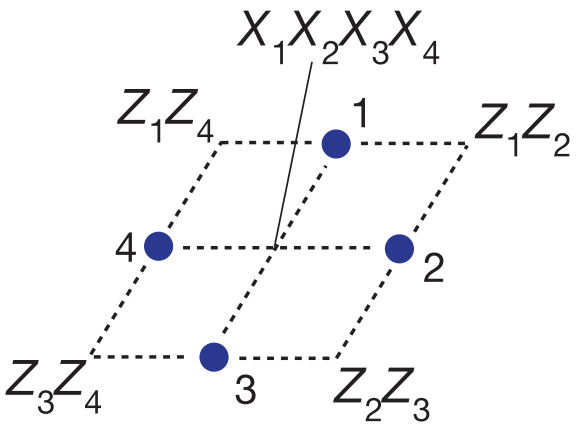


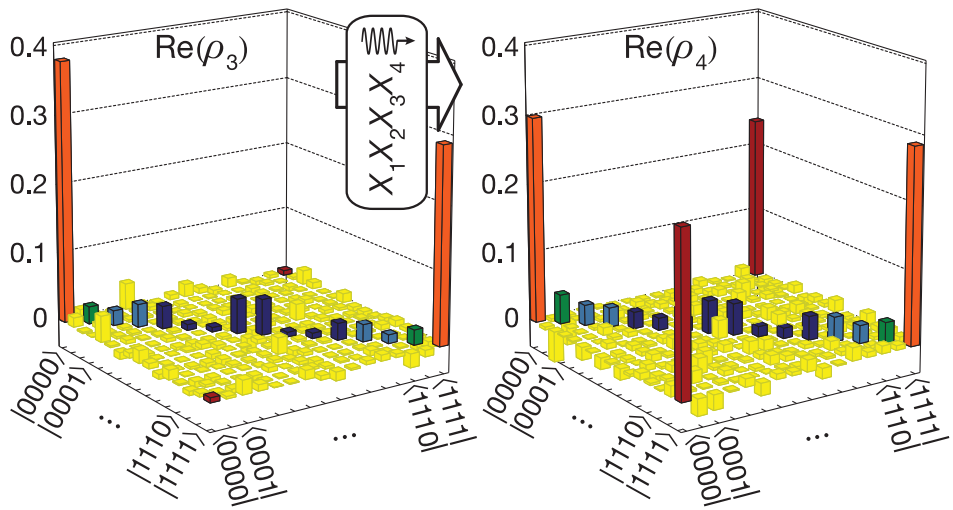
- Proof of concept, the experiment is not scalable
- For efficient quantum simulation we need $n \sim 100$ ions

Quantum Simulation with Ultra Cold Ions [Bar+11]

- Quantum simulation with five trapped ions

  J. T. Barreiro et al. *Nature* **470**, 7335 (2011), pp. 486–491
- Minimal instance of Toric Code stabiliser
- Implements dissipative dynamics through optical pumping





- Proof of concept, the experiment is not scalable
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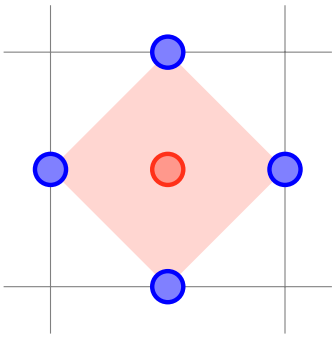
Single Time Step [Wei10; Wei+11; Wei+10]

- Because interactions are local we can focus on single plaquette

$$\mathcal{H} = A_p = \sigma_x^{(1)}\sigma_x^{(2)}\sigma_x^{(3)}\sigma_x^{(4)}$$



- Gate sequence for the simulation consists of four steps:
  - G entangles the control and the target atom
  - $\mathrm{e}^{-\mathrm{i}\phi\sigma_z}$ is the coherent evolution of the control atom, $U(\theta)$ is a controlled spin flip on one ensemble atom
  - G^{-1} reverses the entanglement of control and target atom
  - Optical pumping of the control atom back to $|0\rangle_c$ introduces dissipation

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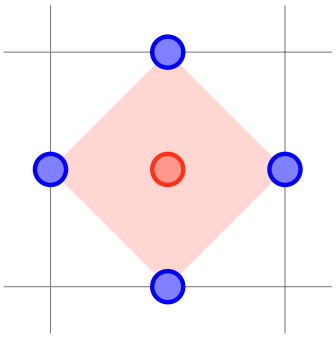

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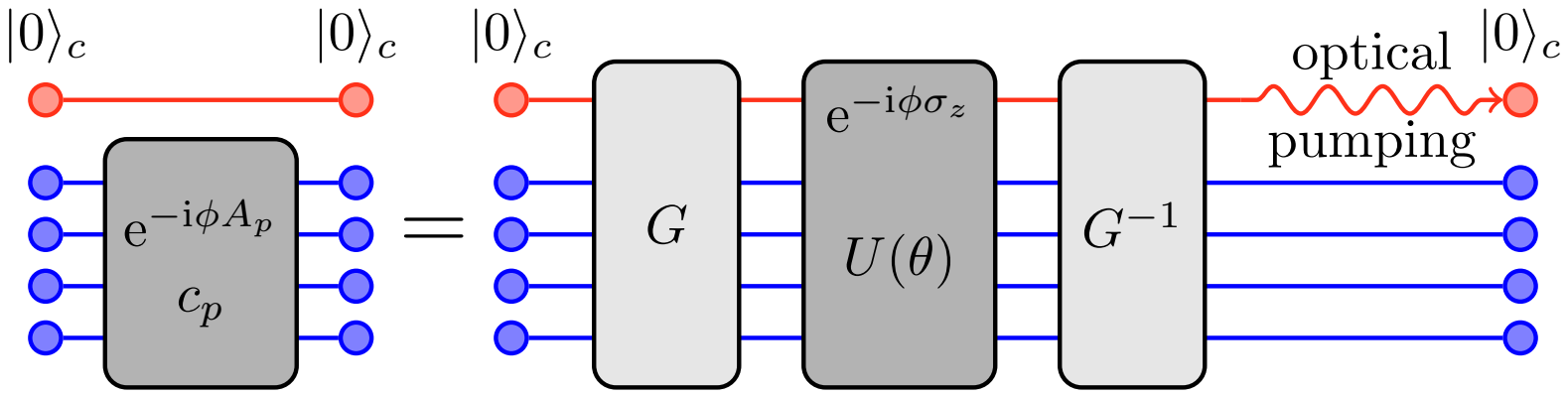
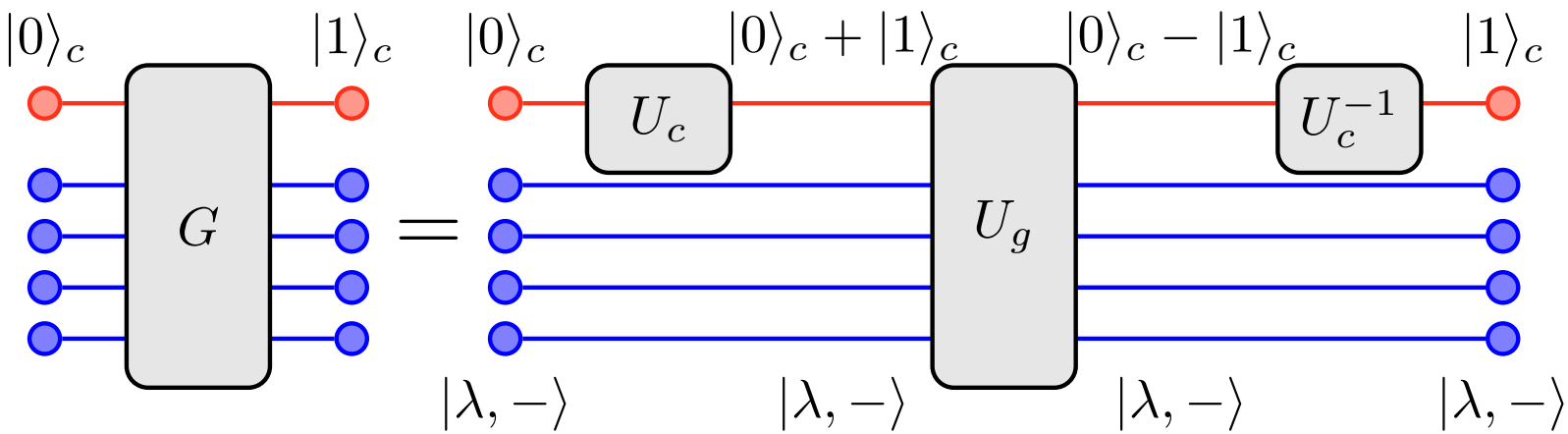


Single Time Step [Wei10; Wei+11; Wei+10]

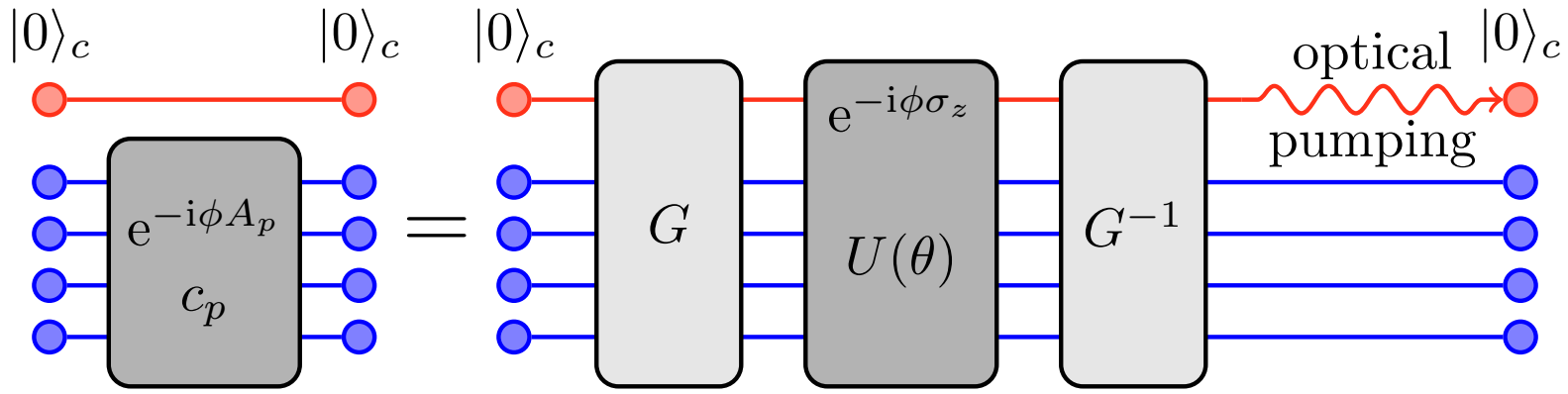
- G is a three step process
  - $U_c = \exp(-\mathrm{i}\pi\sigma_y/4)$ is the standard $\pi/2$ qubit rotation.
  - U_g maps the eigenstate of the ensemble atoms onto the control atom

$$U_g = |0\rangle\langle 0|_c \otimes \mathbb{1} + |1\rangle\langle 1|_c \otimes \prod_{i=1}^{N} \sigma_x^{(i)}$$

  - $U_c^{-1} = \exp(\mathrm{i}\pi\sigma_y/4)$ reverses the rotation



Single Time Step [Wei10; Wei+11; Wei+10]



- G maps the internal state of the ensemble atoms on the control atom

  $G\,|0\rangle_c \otimes |\lambda, +\rangle \to |0\rangle_c \otimes |\lambda, +\rangle$

  $G\,|0\rangle_c \otimes |\lambda, -\rangle \to |1\rangle_c \otimes |\lambda, -\rangle$

  $|\lambda, \pm\rangle$ is eigenstate of the interaction $A_p = \prod_i \sigma_x^{(i)}$ with eigenvalue ± 1

- Phase rotation on the control atom and applying G^{-1} is equivalent to the many-body interaction A_p

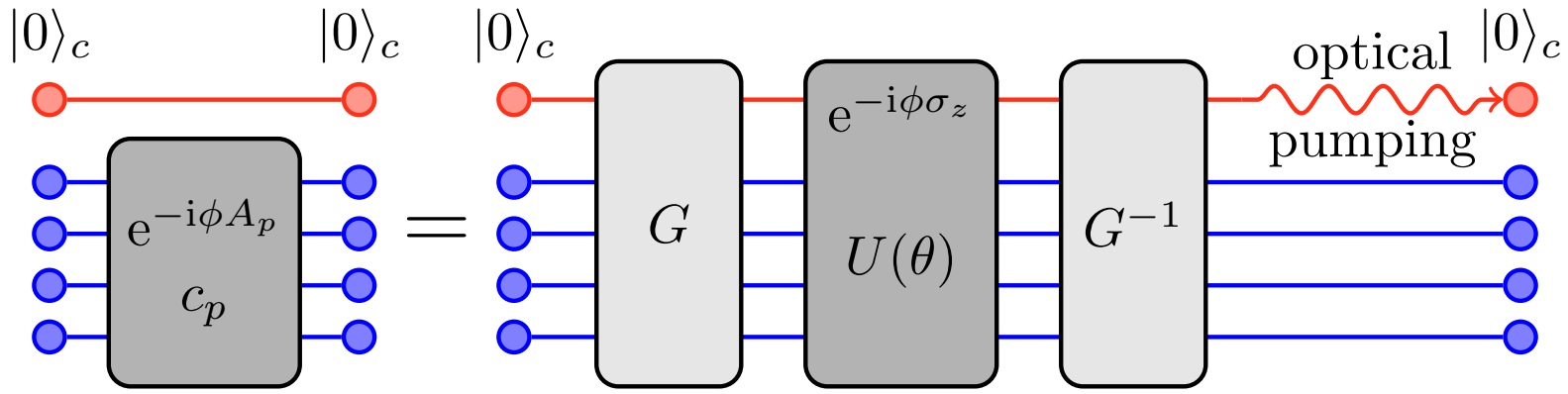
  $$\exp(-i\phi A_p) = G^{-1} \exp(-i\phi\sigma_z^{(c)})\, G$$

- Controlled spin flip onto one of the ensemble atoms

  $U_i(\theta) = |0\rangle\langle 0|_c \otimes \mathbb{1} + |1\rangle\langle 1|_c \otimes \exp(i\theta\sigma_z^{(i)})$

  Leaves $|\lambda, +\rangle$ invariant

Single Time Step [Wei10; Wei+11; Wei+10]



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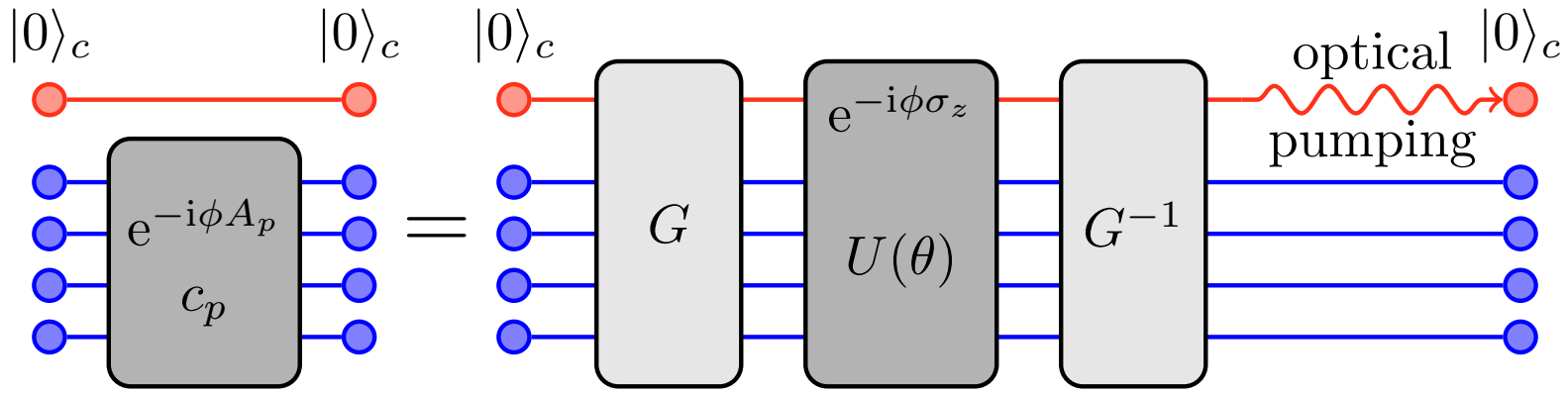

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  Leaves $|\lambda,+\rangle$ invariant

Cooling to the Ground State [Wei10; Wei+11; Wei+10]

- Controlled spin flip onto one of the ensemble atoms

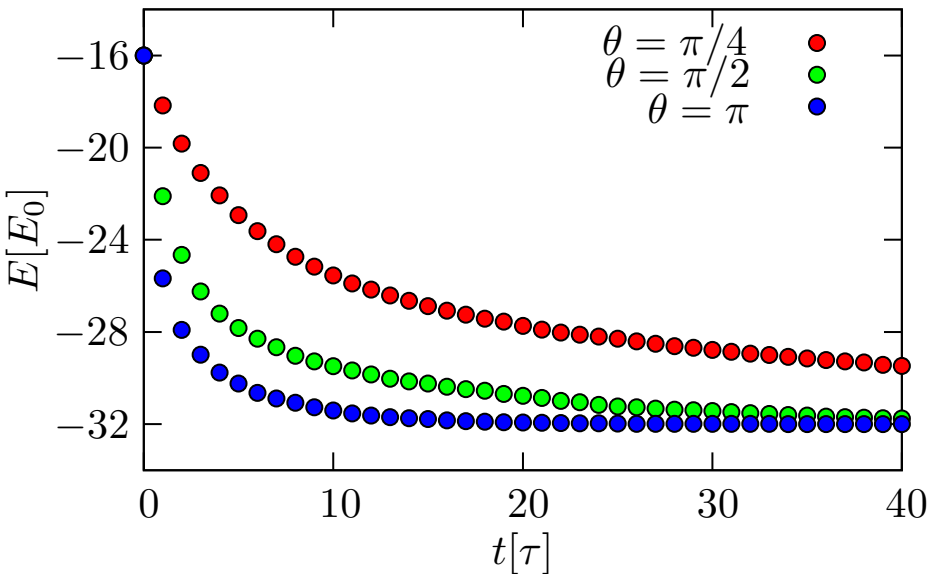
$$U_i(\theta) = |0\rangle\langle 0|_c \otimes \mathbb{1} + |1\rangle\langle 1|_c \otimes \exp(\mathrm{i}\theta\sigma_z^{(i)})$$

  If a flip occurs the control atom is not mapped back to $|0\rangle_c$

- Entanglement is not reversed and whole system evolves according to

$$\partial_t \varrho = \gamma \left(c_i \varrho c_i^\dagger - \frac{1}{2}\{c_i^\dagger c_i, \varrho\} \right) + \mathcal{O}(\theta^3)$$

- Each spin flip moves excitation to adjacent plaquette. For $\theta = \pi$ move takes place with unity probability, i.e. fastest cooling
- Picture: Numerical simulation with 32 particles



↪ [Wei+11]

Fermi-Hubbard Model in 2D [Wei10; Wei+11]

What now about the Hubbard model? Well...

$$
\begin{aligned}
H = &- t \sum_{i,j,\sigma} \left(\sigma^x_{i,j,\sigma} \sigma^x_{i+1,j,\sigma} + \sigma^y_{i,j,\sigma} \sigma^y_{i+1,j,\sigma} \right) \sigma^z_{i',j',\sigma} + t \sum_{i,j,\sigma} \left(\sigma^x_{2i,j,\sigma} \sigma^x_{2i,j+1,\sigma} + \sigma^y_{2i,j,\sigma} \right. \\
&+ t \sum_{i,j,\sigma} \left(\sigma^x_{2i+1,j,\sigma} \sigma^x_{2i+1,j+1,\sigma} + \sigma^y_{2i+1,j,\sigma} \sigma^y_{2i+1,j+1,\sigma} \right) (-1)^{j+1} \sigma^x_{2i'+1,j',\sigma} \sigma^y_{2i'+1,j'} \\
&+ V \sum_{i,j,\sigma} \sigma^z_{2i,2j,\sigma} \sigma^z_{2i+1,2j+1,\sigma} \sigma^x_{2i',2j',\sigma} \sigma^x_{2i'+1,2j',\sigma} \sigma^x_{2i'+1,2j',\sigma} \sigma^x_{2i'+1,2j'+1,\sigma} \\
&+ V \sum_{i,j,\sigma} \sigma^z_{2i+1,2j+1,\sigma} \sigma^z_{2i,2j+2,\sigma} \sigma^x_{2i',2j'+1,\sigma} \sigma^x_{2i'+1,2j'+1,\sigma} \sigma^x_{2i',2j'+2,\sigma} \sigma^x_{2i'+1,2j'+2,\sigma} \\
&+ V \sum_{i,j,\sigma} \sigma^z_{2i+1,2j,\sigma} \sigma^z_{2i+2,2j+1,\sigma} \sigma^y_{2i'+1,2j',\sigma} \sigma^y_{2i'+2,2j',\sigma} \sigma^y_{2i'+1,2j'+1,\sigma} \sigma^y_{2i'+2,2j'+1,\sigma} \\
&+ V \sum_{i,j,\sigma} \sigma^z_{2i+1,2j+2,\sigma} \sigma^z_{2i+2,2j+1,\sigma} \sigma^y_{2i'+1,2j'+1,\sigma} \sigma^y_{2i'+2,2j'+1,\sigma} \sigma^y_{2i'+1,2j'+2,\sigma} \sigma^y_{2i'+2}
\end{aligned}
$$

H. Weimer et al. *Quantum Information Processing* **10**, 6 (2011), pp. 885–906

Fermi-Hubbard Model in 2D [Wei10; Wei+11]

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H = &- t \sum_{i,j,\sigma} \left(\sigma^x_{i,j,\sigma} \sigma^x_{i+1,j,\sigma} + \sigma^y_{i,j,\sigma} \sigma^y_{i+1,j,\sigma} \right) \sigma^z_{i',j',\sigma} + t \sum_{i,j,\sigma} \left(\sigma^x_{2i,j,\sigma} \sigma^x_{2i,j+1,\sigma} + \sigma^y_{2i,j,\sigma} \right. \\
&+ t \sum_{i,j,\sigma} \left(\sigma^x_{2i+1,j,\sigma} \sigma^x_{2i+1,j+1,\sigma} + \sigma^y_{2i+1,j,\sigma} \sigma^y_{2i+1,j+1,\sigma} \right) (-1)^{j+1} \sigma^x_{2i'+1,j',\sigma} \sigma^y_{2i'+1,j'} \\
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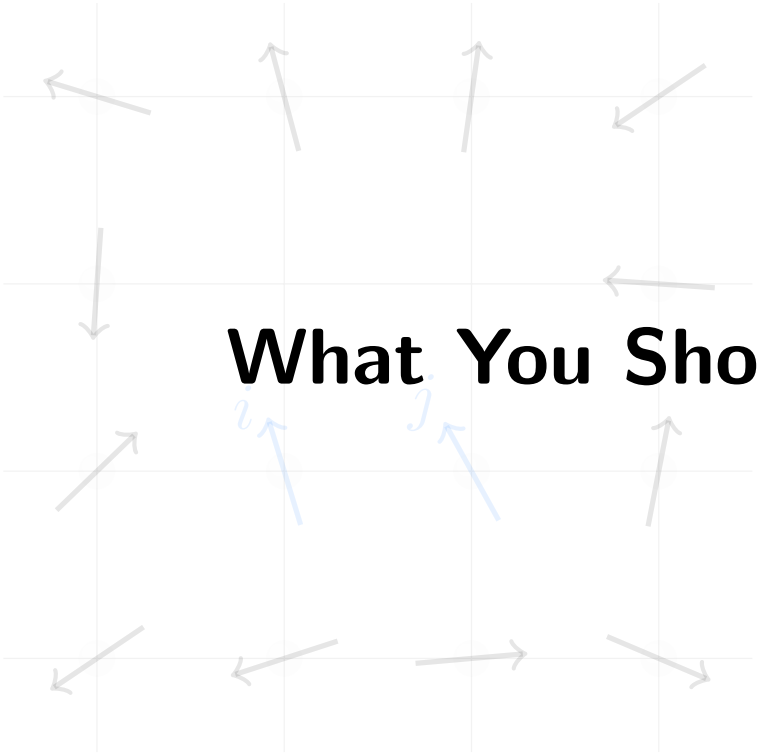
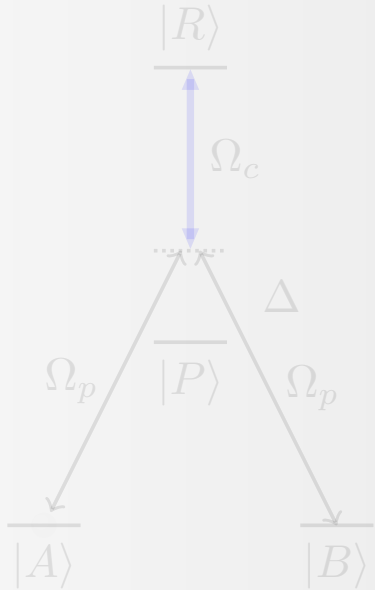
H. Weimer et al. *Quantum Information Processing* **10**, 6 (2011), pp. 885–906

Summary



What You Should Remember



What You Should Remember

- Simulating quantum mechanics on a computer is exponentially hard
- Many-body gates can be used to simulate many-body interactions
- Rydberg atoms are very suitable, because the interactions are long range and allow for single-site addressability
- Dissipative preparation of ground states
- Implementation of complex spin systems
- Toric code can be set up such that it is self correcting

Acknowledgement

ITP3

Przemyslaw Bienias

ITP1

Marcel Klett

Holger Cartarius

PI5

Harald Kübler

References & Further Reading

[Bar+11] J. T. Barreiro et al. *Nature* **470**, 7335 (2011), pp. 486–491.

[BP06] H. P. Breuer and F. Petruccione. *The theory of open quantum systems*. 1st ed. Oxford University Press, 2006.

[Fey82] R. P. Feynman. *Int. J. Theo. Phys.* **21**, 6/7 (1982), pp. 467–488.

[Gaë+09] A. Gaëtan et al. *Nature Physics* **5**, 2 (2009), pp. 115–118.

[Llo96] S. Lloyd. *Science* **273** (1996), pp. 1073–1078.

[Mül+09] M. Müller et al. *Phys. Rev. Lett.* **102**, 17 (2009), p. 170502.

[Sei14] U. Seifert. "Quantenmechanik 2". Lecture. 2014.

[Urb+09] E. Urban et al. *Nature Physics* **5**, 2 (2009), pp. 110–114.

[Wei+10] H. Weimer et al. *Nature Physics* **6**, 5 (2010), pp. 382–388.

[Wei+11] H. Weimer et al. *Quantum Information Processing* **10**, 6 (2011), pp. 885–906.

[Wei10] H. Weimer. "Quantum many-body physics with strongly interacting Rydberg atoms". PhD thesis. 2010.

Important, Experiments

Appendix: Time Scales of the Gate [Mül+09]

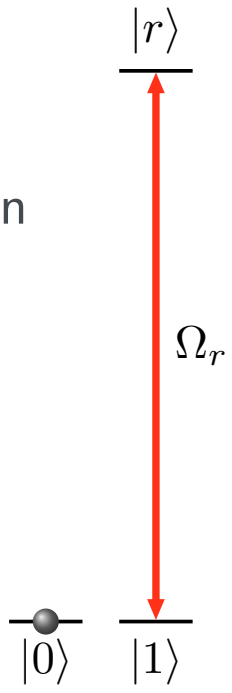
duration of Raman pulse

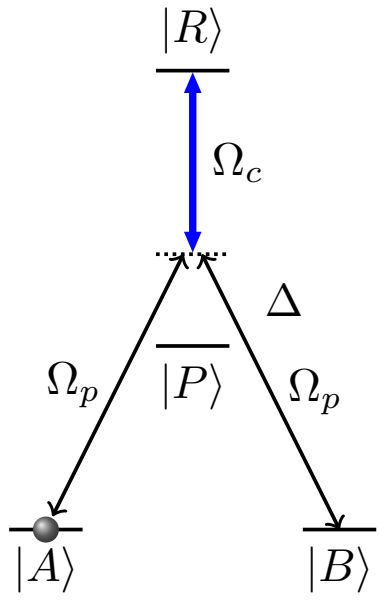
Ryd-Ryd interaction

$$\gamma_{|r\rangle}^{(c)} = \frac{1}{\tau_{|r\rangle}^{(c)}} \ll \frac{1}{T} \sim \frac{\Omega_p^2}{\Delta} \ll \frac{\Omega_c^2}{\Delta} \ll V_{ce}$$

radiative decay of $|r\rangle$

EIT condition





Numbers for ^{87}Rb for a gate fidelity of 99 %:

$$\tau_{|r\rangle}^{(c)} = 66\,\mu\text{s} \qquad \Omega_p = 2\pi \times 70\,\text{MHz}$$

$$T = 0.44\,\mu\text{s} \qquad V_{ce} = 10\Omega_c^2/\Delta \approx 56.3\,\text{GHz}$$

$$\Delta = 2\pi \times 1.2\,\text{GHz} \qquad \Omega_c = 6\Omega_p \approx 2.6\,\text{GHz}$$

Appendix: Decay of a Two-Level System [BP06; Sei14] I

Reminder: Lindblad Master Equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\varrho = -\frac{\mathrm{i}}{\hbar}[\mathcal{H}, \varrho] + \sum_{i=1}^{N^2-1} \gamma_i \left(c_i \varrho c_i^\dagger - \frac{1}{2}\{c_i^\dagger c_i, \varrho\} \right)$$

- Two-Level System

$$\mathcal{H} = \frac{\hbar\omega_0}{2}\sigma_z$$

$$\begin{aligned} c_1 &= \sigma_+, & \gamma_1 &= \gamma_+ \\ c_2 &= \sigma_-, & \gamma_2 &= \gamma_- \\ c_3 &= \sigma_z, & \gamma_3 &= \gamma_z \end{aligned}$$

$$\begin{aligned} \sigma_+ &= |e\rangle\langle g| = \sigma_x + \mathrm{i}\sigma_y \\ \sigma_- &= |g\rangle\langle e| = \sigma_x - \mathrm{i}\sigma_y \end{aligned}$$

Appendix: Decay of a Two-Level System [BP06; Sei14] II

- Master Equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\varrho = -\frac{\mathrm{i}\omega_0}{2}(\sigma_z\varrho - \varrho\sigma_z) + \gamma_+\left(\sigma_+\varrho\sigma_- - \frac{1}{2}\sigma_-\sigma_+\varrho - \frac{1}{2}\varrho\sigma_-\sigma_+\right) + \gamma_-\left(\sigma_-\varrho\sigma_+ - \frac{1}{2}\sigma_+\sigma_-\varrho - \frac{1}{2}\varrho\sigma_+\sigma_-\right) + \gamma_z\left(\sigma_z\varrho\sigma_z - \frac{1}{2}\sigma_z\sigma_z\varrho - \frac{1}{2}\varrho\sigma_z\sigma_z\right) \quad (*)$$

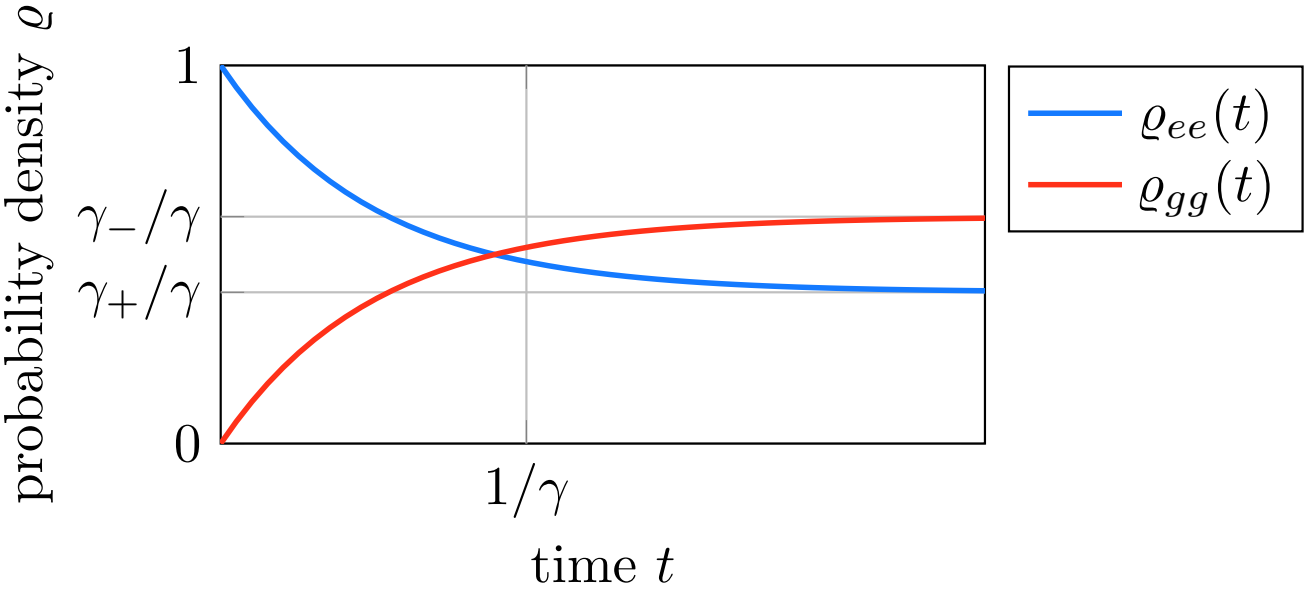
- Time evolution of matrix elements

$$\langle e|(*)|e\rangle: \qquad \frac{\mathrm{d}}{\mathrm{d}t}\varrho_{ee} = \gamma_+\varrho_{gg} - \gamma_-\varrho_{ee}$$

$$\langle g|(*)|g\rangle: \qquad \frac{\mathrm{d}}{\mathrm{d}t}\varrho_{gg} = -\gamma_+\varrho_{gg} + \gamma_-\varrho_{ee}$$

Appendix: Decay of a Two-Level System [BP06; Sei14] III

- Solution of the differential equations



- Calculating the coherences $\langle e|(*)|g\rangle$ and $\langle g|(*)|e\rangle$ allows to derive the principle of detailed balance.